# GALILEAN SYMMETRY IN A NONABELIAN CHERN SIMONS MATTER SYSTEM

**R. Banerjee**[1]
S.N.Bose National Centre for Basic Sciences
Block JD, Sector III, Salt Lake City, Calcutta 700091, India

and

**P. Mukherjee**
A.B.N. Seal College
Cooch Behar, West Bengal
India

**Abstract**

We study the Galilean symmetry in a nonrelativistic model, recently advanced by Bak, Jackiw and Pi, involving the coupling of a nonabelian Chern-Simons term with matter fields. The validity of the Galilean algebra on the constraint surface is demonstrated in the gauge independent formalism. Then the reduced space formulation is discussed in the axial gauge using the symplectic method. An anomalous term in the Galilean algebra is obtained which can be eliminated by demanding conditions on the Green function. Finally, the axial gauge is also treated by Dirac's method. Galilean symmetry is preserved in this method. Comparisions with the symplectic approach reveal some interesting features.

[1]e-mail: narayan@bose.ernet.in

# 1 Introduction

Systems of point particles carrying non-abelian charge interacting with a non-abelian gauge potential have been considered over the last two decades [1]. Similar models in 2+1 dimensions, where the kinetic term of the gauge field is given by the Chern-Simons three form instead of the usual Yang-Mills piece, have been actively investigated in recent years [2-5]. In this context it is interesting to note that it is possible to construct models which are Galilean invariant [6-10] rather than Poincare invariant. This is because the Chern Simons term does not have an elementary photon associated with it so that the Bargmann super-selection rule can be accomodated. Purely Galilean-invariant models are useful to study problems which are difficult when analysed within the full formalism of special relativity.

An important issue in the context of theories involving non-abelian Chern-Simons term is the study of relevant space-time symmetries associated with either Galilean or Poincare transformations. For instance it was claimed [2] that (classical) Poincare covariance gets violated in a theory where the non-abelian Chern-Simons term is coupled to fermions. The calculations were done in the axial gauge which enabled the elimination of the gauge degrees of freedom in favour of the matter variables. Alternatively, it has been shown by one of us [5] that by formulating the model in Dirac's [11] constrained approach which retains all degrees of freedom, the (classical) Poincare covariance is preserved. It is thus clear that the issue of symmetries is rather subtle and requires a thorough and systematic investigation. Indeed since Chern-Simons matter systems are constrained systems, it is possible to discuss different formalisms depending on how one accounts for the constraints. If the results obtained by these distinct formalisms agree, definitive conclusions emerge. Alternatively, if there is disagreement it leads to new findings or fresh insights into the models under investigation. We shall reveal these features explicitly for a non-relativistic model involving the non-abelian Chern-Simons term [4].

In section II we take the nonrelativistic model of Bak,Jackiw and Pi [4] and consider its gauge independent formulation [12-14]. The various space time generators are defined. The closure of the Galilean algebra on the constraint surface is then demonstrated. Sections III and IV comprise the reduced space formalism whereby the gauge freedom associated with the first class constraints is eliminated. In section III this is done in the symplectic approach [4, 15] by choosing the axial gauge. The gauge degrees of freedom are eliminated in favour of the matter degrees by solving the Gauss constraint. The space time generators are defined in the axial gauge. Interestingly, it is found that the complete Galilean algebra does not close in general. Specifically, the bracket of the angular momentum with the hamiltonian does not vanish ; rather it is proportional to a boundary term.

This term vanishes provided some additional restrictions on the Green functions are imposed. The reduced space discussed in section IV, on the contrary, is obtained by following Dirac's [11] constrained analysis. The closure of the full Galilean algebra is demonstrated without any condition, as was necessary in the previous (symplectic) approach. We then discuss the origin of this difference between the two gauge fixed approaches. It is shown to be related to a boundary term involving the square of the connection. This boundary term, incidentally, occurred earlier in the anomalous bracket involving the angular momentum and the hamiltonian in the symplectic approach. Section V contains some concluding remarks.

# 2 Gauge independent formulation of the model

The Lagrangian comprises the Schrodinger Lagrangian minimally coupled with the non abelian Chern-Simons term [4],

$$\mathcal{L} = i\psi^+ D_\circ \psi - \frac{1}{2}(D_i\psi)^+(D_i\psi) - k\epsilon_{\alpha\beta\gamma} tr(A^\alpha \partial^\beta A^\gamma + \frac{2}{3}A^\alpha A^\beta A^\gamma) \tag{1}$$

where the covariant derivative is defined by,

$$D_\mu = \partial_\mu + A_\mu \tag{2}$$

and $A_\mu = A_{\mu a}T^a$ with the antihermitian matrices $T^a$ normalised as,

$$tr(T^a T^b) = -\frac{1}{2}g^{ab} \tag{3}$$

where $g^{ab}$ is the metric [4] in group space. The schrodinger field $\psi$ is an N-component column vector in a certain representation of $T^a$. The Lagrangian (1) can be written in a canonical form by working out the traces,

$$\mathcal{L} = i\psi^+\dot{\psi} - \frac{k}{2}\epsilon_{ij}A_i^a\dot{A}_j^a - \frac{1}{2}(\partial_i\psi^+ - \psi^+ A_i)(\partial_i\psi + A_i\psi) + A_{\circ a}G^a \tag{4}$$

where

$$G^a = i\psi^+ T^a\psi + \frac{k}{2}\epsilon_{ij}(2\partial_i A_j^a + f^{abc}A_i^b A_j^c) \tag{5}$$

Since $\mathcal{L}$ has now been expressed in the desired canonical form, it is simple to read-off the relevant brackets[2] using the symplectic approach [4, 15],

$$\{\psi_n(\vec{x}), \psi_m^*(\vec{x'})\} = -i\delta_{nm}\delta(\vec{x} - \vec{x'}) \tag{6}$$

[2] All brackets are referred to at equal times

$$\{A_i^a(\vec{x}), A_j^b(\vec{x'}\} = \frac{1}{k}\epsilon_{ij} g^{ab} \delta(\vec{x} - \vec{x'}) \tag{7}$$

$$\{A_\circ^a(\vec{x}), \pi_\circ^b(\vec{x'})\} = g^{ab}\delta(\vec{x} - \vec{x'}) \tag{8}$$

where $\pi_\circ$ is the momentum conjugate to $A_\circ$. All other brackets vanish. It is clear from (4) that $A_\circ^a$ is a Lagrange multiplier which enforces the constraint,

$$G^a \approx 0 \tag{9}$$

This constraint is just the analogue of the usual Gauss constraint in electrodynamics, being the generator of the time independent non-abelian gauge transformations. Using the brackets (6-8), it is straightforward to verify this property,

$$\int d^2\vec{x}\alpha_a(\vec{x})\{\psi(\vec{x'}), G^a(\vec{x})\} = \alpha(\vec{x'})\psi(\vec{x'}) \tag{10}$$

$$\int d^2\vec{x}\alpha_b(\vec{x})\{A_i^a(\vec{x'}), G^b(\vec{x})\} = -\partial_i\alpha^a(\vec{x'}) + f^{abc}\alpha_b(\vec{x'})A_{ic}(\vec{x'}) \tag{11}$$

According to Dirac's [11] classification, therefore, $G^a(x)$ is a first class constraint. Indeed it is easy to obtain the involutive algebra,

$$\{G^a(\vec{x}), G^b(\vec{x'})\} = f_c^{ab} G^c(\vec{x})\delta(\vec{x} - \vec{x'}) \tag{12}$$

The equations of motion obtained from (1) are found to be,

$$iD_\circ\psi = -\frac{1}{2}D_i D_i\psi \tag{13}$$

$$\frac{k}{2}\epsilon_{\alpha\beta\gamma}F^{\beta\gamma} = J_\alpha \tag{14}$$

where,

$$J_\circ = T_a J_\circ^a = T_a(-i\psi^+ T^a\psi) \tag{15}$$

$$J_i = T_a J_i^a = -\frac{1}{2}T_a[\psi^+ T^a D_i\psi - (D_i\psi)^+ T^a\psi] \tag{16}$$

are the non-abelian charge density and spatial current density respectively. As usual, the time-component of (14) yields the Gauss constraint (9).

Going over to the hamiltonian formalism we observe that the momenta canonically conjugate to the Lagrange multiplier $A_\circ$ is a constraint,

$$\pi_\circ^a \approx 0 \tag{17}$$

This, together with $G^a$,form the complete set of constraints. The relations,

$$\{\pi^a_\circ(\vec{x}), G^b(\vec{x'})\} = \{\pi^a_\circ(\vec{x}), \pi^b_\circ(\vec{x'})\} = 0 \tag{18}$$

along with (12) constitute the full involutive algebra among the constraints. The canonical hamiltonian is immediately written on inspecting (4),

$$H_c = \int d^2\vec{x}(\frac{1}{2}(D_i\psi)^+(D_i\psi) - A^a_\circ G_a) \tag{19}$$

Using (6-8) it is easy to verify that $H_c$ correctly generates the equations of motion,

$$\partial_\circ\psi_n = \{\psi_n, H_c\} \tag{20}$$

$$\partial_\circ A^a_i = \{A^a_i, H_c\} \tag{21}$$

Let us next discuss the symmetries under various space-time transformations. Consider an infinitesimal transformation,

$$x_\mu \rightarrow x'_\mu = x_\mu + \delta x_\mu \tag{22}$$

$$\phi_n(x) \rightarrow \phi'_n(x') = \phi_n(x) + \delta\phi_n(x) \tag{23}$$

with,

$$\delta x_\mu = \wedge_{\mu\nu}\delta\omega^\nu \tag{24}$$

$$\delta\phi_n = \Phi_{n\nu}\delta\omega^\nu \tag{25}$$

where $\phi_n(x)$ generically denotes the fields in the Lagrangian and $\nu$ can be a single or double index. Then the invariance of any Lagrangian under the above transformations leads to a conserved current [16],

$$J_{\mu\nu} = \frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi_n)}\Phi_{n\nu} - \theta_{\mu\sigma}\wedge^\sigma_\nu \tag{26}$$

where,

$$\theta_{\mu\sigma} = \frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi_n)}\partial_\sigma\phi_n - \mathcal{L}g_{\mu\sigma} \tag{27}$$

With this general input it is straightforward to obtain the various Galileo generators of the present model. For example, under space translations,

$$x_i \rightarrow x'_i = x_i - \delta\omega_i \tag{28}$$

$$x_\circ \to x'_\circ = x_\circ \tag{29}$$

the fields do not transform ($\Phi_{n\nu} = 0$) and the relevant generator is given by

$$\begin{aligned} P_i &= \int d^2\vec{x}\theta_{\circ i}(\vec{x}) \\ &= \int d^2\vec{x}(i\psi^+\partial_i\psi - \frac{k}{2}\epsilon_{kj}A^a_k\partial_i A_{ja}) \end{aligned} \tag{30}$$

Once again using (6-8) the normal transformation properties for the fields may be checked,

$$\{A_k(\vec{x}), P_i\} = \partial_i A_k(\vec{x}) \tag{31}$$

$$\{\psi(\vec{x}), P_i\} = \partial_i\psi(\vec{x}) \tag{32}$$

Similarly, under infinitesimal spatial rotations with angle $\theta$,

$$t' = t \tag{33}$$

$$x'_i = x_i + \theta\epsilon_{ij}x_j \tag{34}$$

the fields transform as,

$$A'_\circ(x') = A_\circ(x), \psi'(x') = \psi(x) \tag{35}$$

$$A'_i(x') = A_i(x) + \theta\epsilon_{ij}A_j(x) \tag{36}$$

Comparing with (24,25) we find,

$$\delta x_i = \theta\epsilon_{ij}x_j, \wedge_{ijk} = \delta_{ij}x_k, A^a_{ijk} = \delta_{ij}A^a_k \tag{37}$$

The rotation generator after an antisymmetrisation now follows from (26),

$$\begin{aligned} J_{ij} &= \int d^2\vec{x}J_{\circ[ij]} \\ &= \int d^2\vec{x}(x_{[i}\theta_{\circ j]} + \frac{k}{2}\epsilon_{[im}A_{ma}A^a_{j]}) \end{aligned} \tag{38}$$

Since there is only one component, we may express this as,

$$J = \int d^2\vec{x}(\epsilon_{ij}x_i\theta_{\circ j} + \frac{k}{2}A_{ja}A^a_j) \tag{39}$$

The basic fields obey covariant transformation laws,

$$\{\psi(\vec{x}), J\} = \epsilon_{ij}x_i\partial_j\psi(\vec{x}) \tag{40}$$

$$\{A^a_i(\vec{x}), J\} = \epsilon_{jk}x_j\partial_k A^a_i(\vec{x}) + \epsilon_{ij}A^a_j(\vec{x}) \tag{41}$$

Finally, we come to the Galileo boosts,

$$x_i \to x'_i = x_i - \vartheta_i t \tag{42}$$

The fields transform as,

$$\psi'(x',t') = \psi(x,t) - i\vartheta_i x_i \psi(x,t) \tag{43}$$

$$A'_i(x',t') = A_i(x,t) \tag{44}$$

$$A'_\circ(x',t') = A_\circ(x,t) + \vartheta_i A_i(x,t) \tag{45}$$

It can be verified that the action corresponding to (1) is invariant under these transformations. Comparing (42-45) with (24,25) yields the correspondence,

$$\wedge_{ij} = -t\delta_{ij}, \Phi_{nj} = -i\psi_n x_j \tag{46}$$

so that the boost generator may be obtained from (26,27) as,

$$\begin{aligned} K_i &= \int d^2\vec{x} J_{\circ i} \\ &= \int (\frac{\partial L}{\partial \dot{\psi}_n} \Phi_{ni} - \theta_{\circ j} \wedge_{ji}) d^2\vec{x} \\ &= tP_i + \int d^2\vec{x} x_i \psi^+ \psi \end{aligned} \tag{47}$$

Under these boosts the basic fields have the usual transformation properties,

$$\{\psi(\vec{x}), K_i\} = t\partial_i \psi(\vec{x}) - ix_i \psi(\vec{x}) \tag{48}$$

$$\{A_j(\vec{x}), K_i\} = t\partial_i A_j(\vec{x}) \tag{49}$$

We have thus shown that the basic fields transform covariantly under all the (Galilean) space-time generators. Consequently it is expected that the complete Galilean algebra ought to be satisfied. Indeed an explicit computation reveals that,

$$\{P_i, P_j\} = \{P_i, H_c\} = \{K_i, K_j\} = 0 \tag{50}$$

$$\{P_i, J\} = \epsilon_{ij} P_j \tag{51}$$

$$\{K_i, J\} = \epsilon_{ij} K_j \tag{52}$$

$$\{P_i, K_j\} = \delta_{ij} \int d^2\vec{x} \psi^+ \psi = \delta_{ij} M \tag{53}$$

$$\{H_c, K_i\} = P_i + \int d^2\vec{x} A_i^a G_a \approx P_i \tag{54}$$

$$\{J, H_c\} = \epsilon_{ij} \int d^2\vec{x} x_i A^a_\circ \partial_j G_a \approx 0 \tag{55}$$

The last two brackets reduce to the conventional result on the constraint surface. Thus, on this surface classical Galilean covariance of the model has been demonstrated. An identical conclusion also holds in the abelian model [10].

The last part of this section is devoted to show that the generators entering in the above (Galilean) algebra are all gauge invariant. In that case these generators can be regarded as physical entities. Using the basic brackets (6-8), the algebra of the Gauss constraint (5) with the various generators may be explicitly calculated to yield,

$$\{P_i, G^a(\vec{x})\} = -\partial_i G^a(\vec{x}) \approx 0 \tag{56}$$

$$\{H_c, G^a(\vec{x})\} = 0 \tag{57}$$

$$\{J, G^a(\vec{x})\} = -\epsilon_{ij} x_i \partial_j G^a(\vec{x}) \approx 0 \tag{58}$$

$$\{K_i, G^a(\vec{x})\} = -t\partial_i G^a(\vec{x}) \approx 0 \tag{59}$$

Thus all the generators are found to be gauge invariant on the constraint surface defined by (9). This completes the gauge independent formulation of the model. The independent canonical pairs are $(A_1, A_2)$ and $(\psi, \psi^+)$. Classical Galilean algebra is satisfied. Furthermore gauge invariance of the relevant generators implies that this algebra should also be preserved in a gauge fixed analysis. Nevertheless it is interesting and instructive to explicitly perform the gauge fixed computations. This will provide fresh insights into the model.

# 3 Gauge fixed formulation : The symplectic approach

The basic idea of this formulation is to work in a reduced space by eliminating the gauge freedom. There are different ways to achieve this target. In the symplectic approach [15] one explicitly solves the Gauss constraint (9) by imposing an additional (gauge) condition, thereby eliminating the gauge degrees of freedom in favour of the matter variables. But whatever gauge condition is chosen, the resulting solution is nonlocal since it involves the inversion of a derivative. In this sense the concept of a local action breaks down. A particularly effective gauge choice is the axial gauge,

$$A^a_1 = 0 \tag{60}$$

since it linearises the Gauss constraint,

$$G^a = k\partial_1 A^a_2 - J^a_\circ = 0 \tag{61}$$

so that the other component of the gauge field is given by,

$$A_2^a = \frac{1}{k} \int d^2\vec{x'} G(\vec{x} - \vec{x'}) J_\circ^a(\vec{x'}) \tag{62}$$

where $G(\vec{x} - \vec{x'})$ is the Green function,

$$\partial_1 G(\vec{x} - \vec{x'}) = \delta(\vec{x} - \vec{x'}) \tag{63}$$

The algebra of the gauge sector is now completely governed by the basic bracket (6) in the matter sector. Using (60) and (62) it follows,

$$\{A_1^a(\vec{x}), A_1^b(\vec{x'})\} = \{A_1^a(\vec{x}), A_2^b(\vec{x'})\} = 0 \tag{64}$$

$$\{A_2^a(\vec{x}), A_2^b(\vec{x'})\} = -\frac{1}{k^2} \int d^2\vec{y} G(\vec{x} - \vec{y}) G(\vec{x'} - \vec{y}) f^{abc} J_{\circ c}(\vec{y}) \tag{65}$$

Likewise it is easy to obtain the algebra of the mixed sector,

$$\{A_1^a(\vec{x}), \psi_n(\vec{x'})\} = 0 \tag{66}$$

$$\{A_2^a(\vec{x}), \psi_n(\vec{x'})\} = \frac{1}{k} (T^a \psi(\vec{x'}))_n G(\vec{x} - \vec{x'}) \tag{67}$$

The algebra involving $A_\circ^a$ is inconsequential since it is a Lagrange multiplier and not a dynamical variable. Note that there is an important subtlety in the solution (62). It does not represent a unique solution for $A_2^a$. There is an arbirtrariness because if $A_2^a(x)$ is a solution then $A_2'^a(x) = A_2^a(x) + f^a(x_\circ, x_2)$ is also a solution. On the other hand there is a residual gauge freedom that survives the axial gauge (60) [5],

$$A_\mu^a(x) \rightarrow A_\mu'^a(x) = A_\mu^a(x) + \partial_\mu \alpha^a(x_\circ, x_2) + f^{abc} A_{\mu b}(x) \alpha_c(x_\circ, x_2) \tag{68}$$

In the abelian theory it is possible, by choosing $f^a = \partial_2 \alpha^a$, to account for the residual gauge freedom and regard (62) as a unique solution for the gauge field. For the nonabelian theory at hand, however, the presence of the extra piece in (68) complicates matters. Indeed if we take,

$$f^a(x_\circ, x_2) = \partial_2 \alpha^a(x_\circ, x_2) + f^{abc} A_{2b}(x) \alpha_c(x_\circ, x_2) \tag{69}$$

we find that while the l.h.s. depends on $(x_\circ, x_2)$ only, the r.h.s. depends on all $x$, so that it becomes impossible to find a solution for $f^a$. The arbitrariness in (62), therefore, persists just as the residual gauge freedom due to (68) remains.

The implementation of a specific gauge is known to modify the manifest covariant transformation of the basic fields [17]. For instance in the radiation gauge the boost law

is found to be altered [17]. In the axial gauge, on the other hand, manifest rotational symmetry is violated. This implies that the transformation (41) under rotations will be modified to preserve the gauge condition (60). Since manifest covariance is spoilt it becomes imperative to verify the Galilean symmetry by working out the algebra (50-55) involving the gauge invariant generators. A detailed analysis shows that apart from one exception the complete Galilean algebra (50-55) is reproduced. The only nontrivial bracket is given by,

$$\{J,H_c\} = \frac{1}{k}\int d^2\vec{x}d^2\vec{y}d^2\vec{z}\{G(\vec{x}-\vec{y})G(\vec{y}-\vec{z})-G(\vec{x}-\vec{z})G(\vec{y}-\vec{z})\}f_{abc}A_2^a(\vec{x})J_2^b(\vec{y})J_\circ^c(\vec{z}) \quad (70)$$

where the current $J_2^b$ and charge density $J_\circ^c$ are defined in (15-16), and $A_2^a$ is given in (62). It is possible to simplify the r.h.s. of (70) by replacing $J_\circ^c$ using (61),

$$\{J,H_c\} = \int d^2\vec{x}d^2\vec{y}d^2\vec{z}\{G(\vec{x}-\vec{y})G(\vec{y}-\vec{z})-G(\vec{x}-\vec{z})G(\vec{y}-\vec{z})\}f_{abc}A_2^a(\vec{x})J_2^b(\vec{y})\partial_1 A_2^c(\vec{z}) \quad (71)$$

Using (63), one can further simplify to obtain,

$$\{J,H_c\} = -\int d^2\vec{x}d^2\vec{y}d^2\vec{z}\partial_1^z\{G(\vec{x}-\vec{z})G(\vec{y}-\vec{z})A_2^c(\vec{z})\}f_{abc}A_2^a(\vec{x})J_2^b(\vec{y}) \quad (72)$$

As pointed out in the previous section the algebra of the gauge invariant generators must be independent of the choice of gauge. Since the complete Galilean algebra was demonstrated earlier, it implies that $\{J,H_c\}$ must vanish in the axial gauge. The r.h.s. of (72) shows that this is not true in general. A simple way to establish compatibility is to demand that the boundary term vanishes, i.e.,

$$\int d^2\vec{z}\partial_1^z\{G(\vec{x}-\vec{z})G(\vec{y}-\vec{z})A_2^c(\vec{z})\} = 0 \quad (73)$$

The above relation gives a restriction on the connection $G(\vec{x}-\vec{y})$. Note that this connection appears squared which must be regularised [4, 18] to make it meaningful. The regularisation must be such that the above condition (73) is satisfied. In that case the complete Galilean algebra is reproduced. It is useful to compare this analysis with Dirac's gauge fixed approach which is given below.

# 4 Gauge fixed formulation : Dirac's approach

In contrast to the symplectic approach the hamiltonian analysis of Dirac [11] distinguishes between first class and second class constraints. The gauge freedom generated by the first

class constraint $G^a \approx 0$ is eliminated by initially choosing a gauge $\chi^b \approx 0$ so that,

$$\det ||\{G^a, \chi^b\}|| \neq 0 \tag{74}$$

Then the complete set of constraints $G^a \approx 0, \chi^b \approx 0$ becomes second class which can be strongly implemented by working with Dirac (star) brackets,

$$\{\phi^a(\vec{x}), \phi^b(\vec{y})\}^* = \{\phi^a(\vec{x}), \phi^b(\vec{y})\} - \int d^2z d^2z' \{\phi^a(\vec{x}), \Omega^c(\vec{z})\}\Omega^{-1}_{cd}(\vec{z}, \vec{z'})\{\Omega^d(\vec{z'}), \phi^b(\vec{y})\} \tag{75}$$

where $\Omega^{-1}_{cd}$ is the inverse of the matrix defined by the Poisson brackets $\{\Omega_c, \Omega_d\}$ of the complete set of constraints $\Omega_c = G_c, \chi_c \approx 0$. The ordinary brackets in (74) merely refer to the fundamental brackets (6-8).

It is worthwhile to highlight some of the fundamental distinctions between the implementation of constraints in the symplectic [4, 15] and Dirac [11] approaches. Contrary to the symplectic case, all the degrees of freedom (either gauge or matter) are retained in the Dirac analysis. There is no need for an explicit solution of the Gauss constraint (61) leading to the non-local structure (62). This also avoids the inherent arbitrariness in the solution (62, 63).

The next step is to compute the Dirac brackets among the basic fields in the axial gauge. The matrix of the Poisson brackets of the constraints is given by,

$$\Omega^{ab}_{ij}(\vec{x}, \vec{y}) = ||\{\Omega^a_i(\vec{x}), \Omega^b_j(\vec{y})\}|| = -\begin{pmatrix} 0 & \partial^x_1 \\ \partial^x_1 & 0 \end{pmatrix} \delta(\vec{x} - \vec{y}) g^{ab} \tag{76}$$

where $\Omega^a_1 = A^a_1 \approx 0$ and $\Omega^a_2 = G^a \approx 0$. The corresponding inverse matrix is found to be,

$$(\Omega^{ab}_{ij}(x, y))^{-1} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} G(\vec{x} - \vec{y}) g^{ab} \tag{77}$$

where the connection has been defined in (63). From the basic brackets (6-8) and using the definition of Dirac brackets in (75) we find the gauge fixed algebra,

$$\{\psi_n(\vec{x}), A^a_2(\vec{y})\}^* = \frac{1}{k} G(\vec{x} - \vec{y})[T^a \psi(\vec{x})]_n \tag{78}$$

$$\{A^a_2(\vec{x}), A^b_2(\vec{y})\}^* = \frac{1}{k} f^{abc} G(\vec{x} - \vec{y})(A_{2c}(\vec{x}) - A_{2c}(\vec{y})) \tag{79}$$

The brackets with $A^a_1(\vec{x})$ vanish as expected from the gauge condition. Note that the second relation preserves the antisymmetry of the bracket under the simultaneous exchange $x \leftrightarrow y, a \leftrightarrow b$.

Let us now compare the Dirac algebra with the corresponding symplectic algebra. Although the bracket (78) agrees with (67), the bracket (79) has a different structure from (65). Thus, at the level of the basic algebra, we find a distinction between the two approaches. It now takes only a little effort to show that the difference between (65) and (79) is just the boundary term in the l.h.s. of (73). Using (61), the bracket (65) reduces to the following,

$$\begin{aligned}
\{A_2^a(\vec{x}), A_2^b(\vec{x'})\} &= -\frac{1}{k}\int d^2\vec{y} G(\vec{x}-\vec{y})G(\vec{x'}-\vec{y}) f^{abc}\partial_1 A_{2c}(\vec{y}) \\
&= \frac{f^{abc}}{k}\int d^2\vec{y}\partial_1^y [G(\vec{x}-\vec{y})G(\vec{x'}-\vec{y})A_{2c}(\vec{y})] \\
&+ \frac{1}{k} f^{abc} G(\vec{x'}-\vec{x})(A_{2c}(\vec{x}) - A_{2c}(\vec{x'})) \\
&= \{A_2^a(\vec{x}), A_2^b(\vec{x'})\}^* + \frac{f^{abc}}{k}\int d^2\vec{y}\partial_1^y [G(\vec{x}-\vec{y})G(\vec{x'}-\vec{y})A_{2c}(\vec{y})] \quad (80)
\end{aligned}$$

where, in going from the first to the second line, we have used (63). Thus, as announced, the difference between the symplectic and Dirac algebras is proportional to the boundary term in (73). If we impose the condition (73) then the two results agree. Finally, using the Dirac brackets (78, 79), it is possible to establish the validity of the complete Galilean algebra (without any restrictions) including the bracket $\{J, H_c\}^*$ which previously yielded an anomalous structure (72) in the symplectic approach. This is not surprising because the anomalous structure in (72) is precisely compensated by the difference in the basic bracket $\{A_2^a, A_2^b\}$ (80) in the two approaches.

# 5 Conclusions

We have investigated by different approaches the (classical) Galilean symmetry in a non-relativistic model involving the coupling of nonabelian Chern-Simons term to matter fields [4]. Since the model is a constrianed system there are different formulations depending on how one accounts for the constraints. A conceptually clean and elegant way of doing this is to work in the gauge independent formalism [12-14]. The various Galilean generators are defined. It is also verified that on the constraint surface, these generators are gauge invariant. The basic fields are found to transform covariantly under the different space-time generators. The Galilean algebra is reproduced on the constraint surface. Since this algebra involves (physical) gauge invariant quantities, it implies that the algebra should be preserved in any gauge fixed compuatation. Two distinct approaches to gauge fixing have been considered in this paper. The first is the symplectic approach [15] whereby the

Gauss constraint is explicitly solved in the axial gauge. The gauge degrees of freedom are eliminated in favour of the matter variables. Since the process involves the inversion of a derivative, the solution for the gauge field is nonlocal. It is found that, except for $\{J, H_c\}$, the Galilean algebra is preserved. The bracket $\{J, H_c\}$ is anomalous; it is in fact proportional to a boundary term involving the square of the connection. To establish compatibility with the Galilean covariance the boundary term ought to vanish which, therefore, imposes restrictions on the connection. The gauge fixed analysis is next repeated in the Dirac [11] formalism. Contrary to the symplectic approach explicit solution of the Gauss constraint is not necessary so that nonlocal expressions do not occur. The complete Galilean algebra (including the result for $\{J, H_c\}$) is valid, without any restrictions on the connection. The difference in the result obtained in the Dirac and symplectic approaches is attributed to the fact that the basic bracket $\{A_2^a(\vec{x}), A_2^b(\vec{x'})\}$ is different in the two approaches. This difference is just proportional to the previously mentioned boundary term occurring in $\{J, H_c\}$ computed in the symplectic approach. Not surprisingly, therefore, the Dirac approach leads to a straightforward validity of the Galilean algebra. It may be worthwhile to consider the effects of ordering so that an analysis of the (quantum) Galilean covariance can be pursued. The gauge independent formulation should be the likely starting point since, contrary to gauge fixed approaches, the algebra in the mixed sector is trivial. Moreover, identification of the independent canonical pairs $(A_1^a, A_2^a), (\psi, \psi^*)$ is also clean. These and other related issues are currently under investigation.

# References


[1] S.K. Wong, Nuovo Cimento A 65, 689 (1970).

[2] C.R. Hagen, Phys. Rev. D 31, 331 (1985).

[3] G. Dunne, R. Jackiw, S. -Y. Pi, and C.A. Trugenberger, Phys. Rev. D 43, 1332 (1991); 45, 3012(E) (1992); G. Dunne, Comm. Math. Phys. 150, 519 (1993).

[4] D. Bak, R. Jackiw and S. -Y. Pi, Phys. Rev. D 49, 6778 (1994).

[5] R. Banerjee and B. Chakraborty - Hamiltonian formulation of the theory with a non-abelian Chern-Simons term coupled to fermions, To appear in Ann. Phys (N.Y.).

[6] C.R. Hagen, Ann. Phys. (N.Y.) 157, 342 (1984); Phys. Rev. D 31, 2135 (1985).

[7] R. Jackiw, Ann. Phys. (N.Y.) 201, 83 (1990).

[8] R. Jackiw and S. -Y. Pi, Phys. Rev. D 42, 3500 (1990); 48, 3929(E) 1993.

[9] R. Jackiw and S. -Y. Pi, Phys. Rev. Lett. 64, 2969 (1990); 66, 2682 (1991).

[10] R. Banerjee and B. Chakraborty, Phys. Rev. D 49, 5431 (1994); B. Chakraborty and A.S. Majumdar, On Galilean covariance in a non-relativistic model involving a Chern-Simons term. To appear in Ann. Phys. (N.Y.).

[11] P.A.M. Dirac, Lectures on Quantum Mechanics (Belfer Graduate School of Science, Yeshiva University, New York, 1964).

[12] R. Banerjee, Phys. Rev. Lett. 69, 17 (1992); ibid. 70, 3519 (1993); Phys. Rev. D 48, 2905 (1993).

[13] R. Banerjee, Nucl. Phys. B 390, 681 (1993); ibid, B 419, 611 (1994).

[14] B. Chakraborty, Ann. Phys. (N.Y.) 244, 312 (1995).

[15] L. Faddeev and R. Jackiw, Phys. Rev. Lett. 60, 1692 (1988).

[16] L.H. Ryder, Quantum Field Theory (Cambridge Univ. Press, Cambridge, 1985) Ch.3.

[17] A. Hanson, T. Regge and C. Teitelboim, "Constrained Hamiltonian Systems" (Academia Nazionale dei Lincei, Rome, 1976).

[18] A. Kapustin and P. Pronin, Phys. Lett. B 303, 45 (1993).